Since only the z- component of A is non-zero

$$\frac{\partial A_z}{\partial z} = -\frac{1}{c^2}\frac{\partial \phi}{\partial t}$$

therefore

$$-\frac{1}{c^2}\frac{\partial \phi}{\partial t} = \frac{\mu_0}{4\pi}\dot{p}(t)\frac{\partial}{\partial r}\left(\frac{e^{-ikr}}{r}\right)\cos\theta$$

$$\left(\because \frac{\partial r}{\partial z} = \cos\theta\right)$$

i.e. $$\frac{\partial \phi}{\partial t} = -\frac{1}{4\pi\varepsilon_0}\dot{p}(t)\cos\theta\frac{\partial}{\partial r}\left(\frac{e^{-ikr}}{r}\right)$$

$$\left[\because c^2 = \frac{1}{\mu_0\varepsilon_0}\right]$$

$$\text{i.e. } \phi = -\frac{1}{4\pi\varepsilon_0}\dot{p}(t)\cos\theta\frac{\partial}{\partial r}\left(\frac{e^{-ikr}}{r}\right) \quad --(21)$$

We have obtained A and ϕ.

We can now determine E and B

In spherical polar coordinates, the components are

$$A_r = \frac{\mu_0}{4\pi r} i\omega p_0 \cos\theta \exp\{i\omega(t - r/c)\} = \frac{\mu_0}{4\pi r}\dot{p}\cos\theta\, e^{-ikr}$$

$$= \frac{\mu_0}{4\pi r}\dot{p}\cos\theta\, e^{-ikr}$$

$$A_\theta = \frac{\mu_0}{4\pi r} i\omega p_0 \sin\theta \exp\{i\omega(t - \frac{r}{c})\} = \frac{\mu_0}{4\pi r}\dot{p}\sin\theta\, e^{-ikr}$$

$$--(22)$$

$$A_\phi = 0$$

The components of E and H are

$$E_r = -\frac{\partial \phi}{\partial r} - \frac{\partial A_r}{\partial t} = \frac{ipk}{2\pi\varepsilon_0 r}\cos\theta\left[1 - \frac{i}{kr}\right]\frac{e^{-ikr}}{r}$$

$$E_\theta = -\frac{1}{r}\frac{\partial \phi}{\partial \theta} - \frac{\partial A_\theta}{\partial t} = -\frac{pk^2}{4\pi\varepsilon_0}\sin\theta\left[1 - \frac{i}{kr}\left(1 - \frac{i}{kr}\right)\right]\frac{e^{-ikr}}{r}$$

$$--(23)$$

$$E_\phi = 0$$

$$H_r = 0,\ H_\theta = 0,\ H_\phi = -\frac{cpk^2}{4\pi}\sin\theta\left[1 - \frac{i}{kr}\right]\frac{e^{-ikr}}{r} \quad --(24)$$

Eq. (23) and (24) are Hertz's relations for the oscillating dipole.

We may imagine the space divided into two regions:

(i) The region in which $|r| \leq \lambda$, near zone. field in this case is exactly the same as that for an electrostatic dipole.

(ii) The region in which $|r| \geq \lambda$, radiation zone. The field in this region vary as $\frac{1}{r}$.

We examine the relations (23) and (24) in the near zone and the radiation zone.

(i) Near zone: $|r| \leq \lambda$, i.e. $kr << 1$

$$E_r \simeq \frac{p\cos\theta}{2\pi\varepsilon_0 r^3}, \quad E_\theta \simeq \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}, \quad E_\phi = 0 \quad -(25)$$

$$H_r = 0, \quad H_\theta = 0, \quad H_\phi \simeq \frac{i\omega p\sin\theta}{4\pi r^2}$$

These relations are equivalent to the field of an electromagnetic dipole, the ratio of magnetic to electric field in this zone is

$$\frac{\omega\mu_0|H|}{k|E|} \simeq \frac{\omega\mu_0}{k}\varepsilon_0\omega r = kr << 1$$

Electric field dominates in this zone.

(ii) Radiation zone: $kr >> 1$

$$E_r = 0, \quad E_\theta = -\frac{pk^2}{4\pi\varepsilon_0}\sin\theta\frac{e^{-ikr}}{r}, \quad E_\phi = 0$$

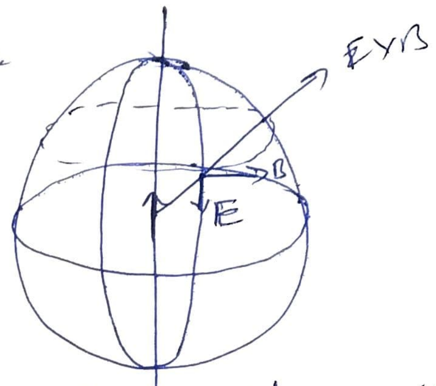
$$H_r = 0, \quad H_\theta = 0, \quad H_\phi = -\frac{cpk^2}{4\pi}\sin\theta\frac{e^{-ikr}}{r} \quad -(26)$$

[neglecting the terms involving $\frac{1}{r^2}$ and higher orders]

From eqn. (26)

$$E_\theta = \sqrt{\frac{\mu_0}{\varepsilon_0}}\, H_\phi \quad --(27)$$

The direction of fields are shown as;



The fields are mutually perpendicular for the radiation zone $E_r = 0$, hence, the field distribution is spherical with the electric field as circles of longitude and magnetic field as the circles of Latitude.

The field is maximum at the equator and zero at the poles. Since the magnetic field is transverse everywhere, the radiation from an oscillating dipole is generally TM.

At large distances $E_r \to 0$ and the electric field is also transverse, hence the waves are TEM.

Power of electromagnetic radiation and its polarization for an oscillating dipole

Let us calculate the instantaneous power W crossing the surface of a sphere of radius say r, in the radiation zone

The time average Poynting vector $\langle N \rangle$ is

$$\langle N \rangle = \frac{1}{2}\, \text{Re}\,(E \times H^*)$$